

Semester Two Examination, 2020

Question/Answer booklet

**MATHEMATICS
SPECIALIST
UNITS 1&2**

**Section Two:
Calculator-assumed**

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (98 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(6 marks)

- (a) Triangle ABC has vertices $A(2, -3)$, $B(2, 5)$ and $C(12, -1)$. Determine the area of this triangle after it has been transformed using the matrix $\begin{bmatrix} -4 & 4 \\ 3 & 3 \end{bmatrix}$. (3 marks)

- (b) Show use of matrix algebra, including the coefficients of any inverse matrix used, to solve the following system of linear equations: (3 marks)

$$2a + 3b = 55$$

$$4a + 5b = 79$$

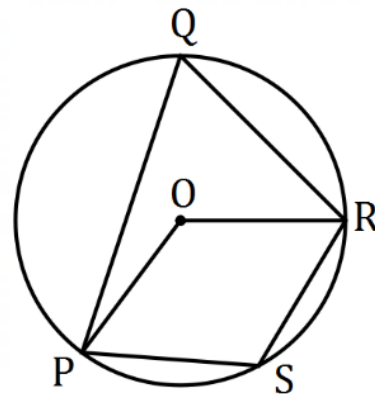
Question 10

(6 marks)

- (a) Prove that the opposite angles of a cyclic quadrilateral are supplementary. (3 marks)

- (b) The points P, Q, R and S lie on the circle with centre O so that $PS = RS$ and $\angle PQR = 48^\circ$.

Determine the size of $\angle ORS$.



(3 marks)

Question 11

(8 marks)

Two vectors are $\mathbf{p} = \begin{pmatrix} 72 \\ -154 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -39 \\ 252 \end{pmatrix}$. Determine

- (a) the magnitude of \mathbf{p} . (1 mark)
- (b) the angle between the directions of \mathbf{q} and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. (2 marks)
- (c) the value of the scalar constant k so that $18\mathbf{p} + k\mathbf{q}$ is parallel to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. (2 marks)
- (d) a vector \mathbf{r} that is perpendicular to \mathbf{p} with the magnitude of \mathbf{q} . (3 marks)

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Question 12

(8 marks)

The vertices of triangle T are $A(2, 3)$, $B(-5, 1)$ and $C(0, 12)$.

Transformation M is a translation by vector $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

- (a) State the coordinates of the image of C after triangle T is transformed by M . (1 mark)

Transformation N is a reflection in the line $x + y = 0$.

- (b) Determine the transformation matrix for N and state the coordinates of the image of A after triangle T is transformed by M and then by N . (3 marks)

Transformation P is a rotation of 135° clockwise about the origin.

- (c) Determine the exact coordinates of the image of B after triangle T is transformed by N and then by P . (3 marks)

- (d) Write a matrix expression for the transformation matrix Q that represents the inverse of transformation P followed by the inverse of transformation N . There is no need to simplify your expression. (1 mark)

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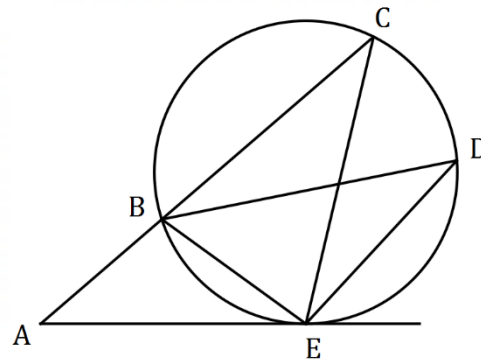
Question 13

(8 marks)

- (a) In the diagram shown (not to scale) ABC is a straight line and B, C, D and E lie on a circle.

AE is a tangent to the circle at E ,
 $\angle BEC = 76^\circ$ and $\angle BDE = 27^\circ$.

Determine, with reasons, the size of $\angle BAE$.

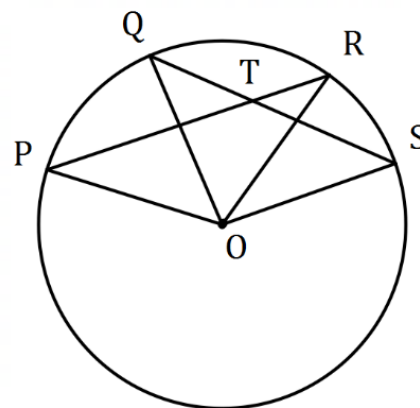


(4 marks)

- (b) In the diagram shown (not to scale) P, Q, R and S lie on a circle centre O and chords QS and PR intersect at T .

$\angle POQ = 42^\circ$ and $\angle ROS = 35^\circ$.

Determine, with reasons, the size of $\angle RTS$.



(4 marks)

Question 14

(8 marks)

(a) State whether each of the following statements is true or false, supporting each answer with an example or counterexample.

(i) $\forall a, b, c, d \in \mathbb{R}$, if $a < b$ and $c < d$ then $ac < bd$. (2 marks)

(ii) $\forall n \in \mathbb{N}$, if n is even then $3^n - 2$ is prime. (2 marks)

(b) Prove by contradiction that $ABCD$ is not a cyclic quadrilateral if diagonal AC of length 13 cm cuts diagonal BD of length 12 cm at E so that $AE = DE = 4$ cm. (4 marks)

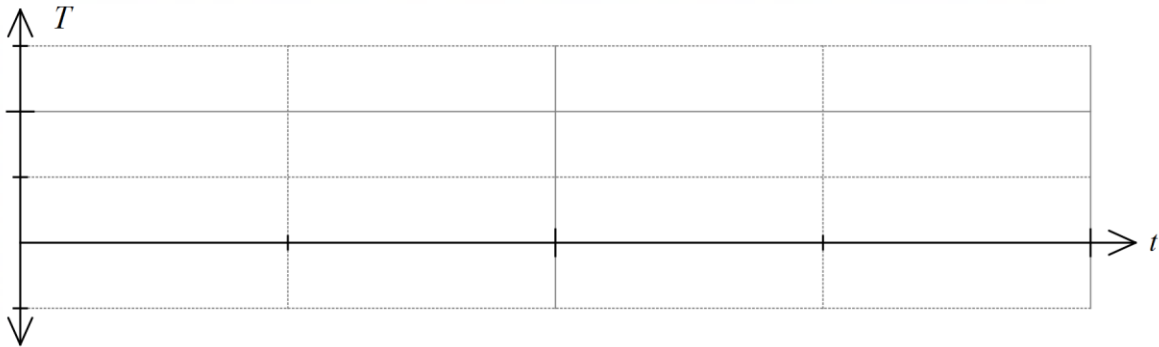
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Question 15

(8 marks)

Starting at midnight ($t = 0$), the temperature T at a resort was observed to vary sinusoidally over the course of a day, reaching a high of 28.7°C at 2 pm after a low of -5.9°C at 2 am. Let t be the time in hours from midnight.

- (a) Use the above information to **sketch** a graph showing how T varies with t during the day. (2 marks)



- (b) Determine an algebraic model for T as a function of t . (4 marks)

- (c) Use your model to determine the proportion of the day that the temperature at the resort was below 4°C . (2 marks)

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Question 16

(8 marks)

(a) Determine the number of integers between 1 and 499 that are

(i) divisible by 56. (1 mark)

(ii) divisible by 7 or by 8 but not by 56. (3 marks)

(b) A playlist offered by a music streaming service has 22 different songs. Every time a playlist is streamed, the songs are shuffled into a random arrangement.

Show that after the playlist has been streamed 30 000 times, at least 4 of those streams began with the same 3 songs in the same order. (4 marks)

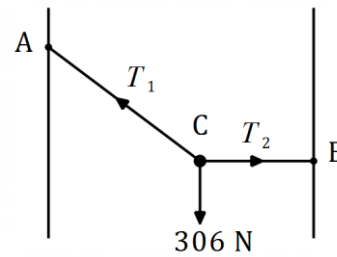
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Question 17

(8 marks)

A small object C of weight 306 N is suspended above level ground and between two vertical walls by two strings. The walls are 192 cm apart.

Point A lies on one wall so that string AC is 185 cm long and point B lies on the other wall so that string BC is horizontal and 88 cm long.



- (a) Determine the tension T_1 in string AC .

(3 marks)

- (b) String BC is lengthened so that the height of C above the ground decreases by 29 cm and $\angle ACB = 90^\circ$. Determine the tension T_2 in string BC .

(5 marks)

Question 18

(6 marks)

- (a) Given that $A = \begin{bmatrix} a-3 & 8 \\ 2a+1 & 3-a \end{bmatrix}$, determine the value(s) of the real constant a so that A is its own inverse. (3 marks)

- (b) Let $B = \begin{bmatrix} -1 & 5 \\ 2 & -8 \end{bmatrix}$ and $C = \begin{bmatrix} 7 \\ -11 \end{bmatrix}$. Determine X when $X - 5BC = B^2X$. (3 marks)

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Question 19

(8 marks)

(a) 7 students from Class A, 9 from Class B and 5 from Class C have nominated for the 3 places available in the team for a mathematics competition. Determine the number of different teams that can be formed if

(i) the students are chosen from the same class. (2 marks)

(ii) at least 2 students in the team are chosen from Class B. (2 marks)

(b) Prove that for $n \geq 4$, ${}^nC_3 + {}^nC_4 = {}^{n+1}C_4$. (4 marks)

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Question 20

(8 marks)

A common proof that $\sqrt{3}$ is irrational begins by assuming that $\sqrt{3}$ is rational, so that $\sqrt{3} = \frac{a}{b}$.

- (a) Describe two properties of variables a and b that the proof requires, other than $b \neq 0$.
(2 marks)

The next step obtains the relationship $a^2 = 3b^2$, from which it is deduced that $a = 3A, A \in \mathbb{Z}$.

- (b) Prove, using the contrapositive, that if a^2 is a multiple of 3 then so is a .
(4 marks)

- (c) Complete the proof that $\sqrt{3}$ is irrational.
(2 marks)

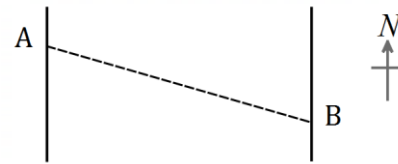
Question 21

(8 marks)

Points A and B lie on opposite sides of a river so that B is 240 m away from A on a bearing of 105° .

A uniform current flows due north in the river between A and B at 0.32 m/s.

Sam can swim at a steady speed of 1.2 m/s and plans to swim from A to B and then back to A .



- (a) Determine the bearing Sam should swim to move directly towards B from A . (3 marks)

- (b) Show that Sam takes 30 seconds less to swim the return leg than the first leg. (5 marks)

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Supplementary page

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